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9. An analog, oligomer-based method for determining a mathematical result of

carrying out an operation of matrix algebra on input data,

wherein single-stranded oligomers  $E_i$  and  $\underline{E}_i$  are a subset of all single-stranded oligomers and are each in 1:1 correspondence with the basis vectors  $e_i$ ,  $i = 1, 2, \dots, m$  in an abstract  $m$ -dimensional vector space;

wherein a set of the oligomers  $E_i$  and  $\underline{E}_i$  represents an  $m$ -component vector  $V = \sum_i V_i e_i$ ,

wherein the  $E_i$  and  $\underline{E}_i$  oligomers have complementary nucleotide sequences, with the  $E_i$  oligomers representing the  $i$ -th component of  $V$  for which the amplitude  $V_i$  is positive, and the  $\underline{E}_i$  oligomers representing the  $i$ -th component of  $V$  for which  $V_i$  is negative; and

wherein the concentration of each of the oligomers  $E_i$  or  $\underline{E}_i$  is proportional to the absolute value of the amplitude  $V_i$  of the  $i$ -th component of  $V$ ,

the method comprising the steps of

(1) obtaining a composition comprising at least one set of single-stranded oligomers  $E_i$  and  $\underline{E}_i$  representing the components of a vector, wherein the concentrations of the oligomers  $E_i$  or  $\underline{E}_i$  in the composition are proportional to the absolute values of the amplitudes of the components they represent, which composition represents input data; and

(2) subjecting said composition to at least one physical or chemical treatment having an effect on said oligomers in said composition that is an analog representation of an operation of matrix algebra, and

(3) detecting the effect of said treatment on said oligomers in said composition to determine the analog result of carrying out said operation of matrix algebra on said input data.

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10. The method of claim 9, wherein the oligomers independently comprise subunits selected from the group consisting of deoxyribonucleotides, ribonucleotides, and analogs of deoxyribonucleotides or ribonucleotides; and any single oligomer comprises one or a combination of two or more of said different types of subunits.

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11. The method of claim 10, wherein said at least one physical or chemical treatment in step (2) is selected from the group consisting of (a) changing the <sup>relative</sup> concentration of the oligomers in said composition, (b) allowing complementary oligomers in said composition to hybridize to each other, (c) determining the concentration of double-stranded oligomers in the composition, (d) separating double-stranded oligomers from non-double-stranded oligomers in the composition, (e) measuring the rate of hybridization of complementary oligomers in the composition, (f) ligating oligomers together, (g) adding oligomer subunits to an end of an oligomer in an enzyme-catalyzed reaction, (h) using an oligomer as a template in synthesizing a complementary oligomer sequence in a polymerase-catalyzed reaction, (i) <sup>phosphorylating or de-phosphorylating</sup> ~~modifying~~ a terminus of an oligomer in an enzyme-catalyzed reaction ~~that does not add an additional oligomer subunit,~~ and (j) cleaving an oligomer with a restriction enzyme.

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12. The method of claim 11 wherein said operation of matrix algebra is multiplication of a vector by a scalar, and said method comprises changing the total concentration of said oligomers in said composition by a factor equivalent to the scalar by which the vector is multiplied, thereby obtaining an oligomer-containing composition that represents the product of multiplying said vector by said scalar.

13. The method of claim 11 wherein said operation of matrix algebra is addition of

vectors, and

sub C74 } said method comprises obtaining, for each vector to be added, a set of single-stranded oligomers  $E_i$  and  $\underline{E}_i$  representing the components of the vector, wherein the concentrations of the oligomers  $E_i$  and  $\underline{E}_i$  are proportional to the absolute values of the amplitudes of the components they represent;

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allowing complementary oligomers in the resulting mixture to hybridize; and

separating the fully hybridized, double-stranded oligomers from the resulting mixture of oligomers, thereby obtaining a set of non-double-stranded oligomers that represents the sum of the added vectors.

14. The method of claim 11 wherein said operation of matrix algebra is determining the inner product of two vectors  $V_i$  and  $W_i$ , and said method comprises:

Sub D1 } (i) obtaining for each vector  $V_i$  and  $W_i$ , a set of single-stranded oligomers  $E_i$  and  $\underline{E}_i$  representing the components of the vector, wherein the concentrations of the oligomers  $E_i$  and  $\underline{E}_i$  are proportional to the absolute values of the amplitudes of the components they represent; and

combining a sample of the oligomers representing vector  $V_i$  with a sample of the oligomers representing vector  $W_i$ , and measuring both the rate of hybridization  $R$  and the concentration of double-stranded oligomers present in the mixture following hybridization;

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(ii) obtaining for one of said vectors  $V_i$  or  $W_i$  an isolated set of single-stranded oligomers  $\underline{V}_i$  or  $\underline{W}_i$ , respectively, that are complementary to the set of oligomers representing said vector  $V_i$  or  $W_i$ , respectively, wherein the relative concentrations of said complementary oligomers in  $\underline{V}_i$  or  $\underline{W}_i$  are proportional to the relative concentrations of the oligomers in  $V_i$  or  $W_i$ , respectively, to which they are complementary, and

combining a sample of said set of complementary oligomers  $\underline{V}_i$  or  $\underline{W}_i$ , with a sample of the other of said vectors,  $W_i$  or  $V_i$ , respectively and measuring both the rate of hybridization  $R_+$ , and the concentration of double-stranded oligomers present in the mixture following hybridization; and

(iii) taking the difference between the values  $R_+$  and  $R_-$  after normalizing said values with respect to oligomer concentration, thereby obtaining a numerical value proportional to the inner product of the two vectors.

15. The method of claim 11 wherein said operation of matrix algebra is obtaining the outer product matrix of two vectors  $V_i$  for  $i = 1, 2, \dots, m$ , and  $W_j$  for  $j = 1, 2, \dots, n$ , and

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said method comprises obtaining a set of single-stranded oligomers, each of which comprises (i) a first single-stranded oligomer sequence selected from the group consisting of  $E_i$  or  $\underline{E}_i$  for each  $i$ -th component of  $V$  for  $i = 1, 2, \dots, m$ , and (ii) a second single-stranded oligomer sequence selected from the group consisting of  $E_j$  or  $\underline{E}_j$  for each  $j$ -th component of  $W$  for all  $j = 1$  to  $j = n$ ,

wherein said resulting set of single-stranded, dimeric oligomers is an analog representation of the matrix formed as the outer product of said two vectors.

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16. The method of claim 11 wherein said operation of matrix algebra is obtaining the inner product of a matrix and a vector, and

said method comprises

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(a) obtaining a set of single-stranded oligomers representing matrix  $T$ , wherein each matrix component  $T_{ij}$  is represented by single-stranded oligomers comprising a dimeric oligomer sequence selected from the group consisting of  $5' - \{E_i\} \{E_j\} - 3'$ ,  $5' - \{E_i\} \{E_j\} - 3'$ ,  $5' - \{E_i\} \{E_j\} - 3'$ , and  $5' - \{E_i\} \{E_j\} - 3'$ , and wherein the concentrations of said dimeric oligomers  $T_{ij}$  are proportional to the absolute values of the amplitudes  $X_i$  of the matrix components they represent;

(b) obtaining a set of single-stranded oligomers  $E_i$  and  $E_j$  representing the components of a vector  $V$ , wherein the concentrations of said oligomers  $E_i$  and  $E_j$  are proportional to the absolute values of the amplitudes  $V_i$  of the vector components they represent;

(c) obtaining a set of single-stranded oligomers  $E_i$  and  $E_j$  having the sequences of the 5' portions of said dimeric oligomers representing matrix  $T_{ij}$  which also comprise in their 3' portions said sequences representing said vector  $V$ ,

wherein the relative concentrations of said oligomers in said set of oligomers having the 5' sequences of said dimeric  $T_{ij}$  oligomers are proportional to the relative concentrations of the oligomers in the 3' portions of the corresponding dimeric  $T_{ij}$  oligomers having the same sequences as said sequences representing vector  $V$ ;

(d) obtaining a set of single-stranded oligomers  $E_i$  and  $E_j$  complementary to the sequences of the 5' portions of said dimeric oligomers representing matrix  $T_{ij}$  which also comprise in their 3' portions  $E_i$  or  $E_j$  sequences complementary to said sequences representing vector  $V$ ,

wherein the relative concentrations of said oligomers in said set of oligomers complementary to the 5' sequences of said dimeric  $T_{ij}$  oligomers are proportional to the relative

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concentrations of said oligomers in the 3' portions of the corresponding dimeric  $T_{ij}$  oligomers that are complementary to said sequences representing vector  $V$ ; and

(e) combining said set of single-stranded oligomers obtained in step (c) with said set of single-stranded oligomers obtained in step (d), to obtain a set of single-stranded oligomers that is an analog representation of the inner product of said matrix  $T$  and said vector  $V$ .

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17. A method for obtaining a data set  $V_i^b$  from an oligomer-based, content-addressable memory following input of a data set  $U_i^b$  that represents a portion of  $V_i^b$ , wherein data elements in the form of m-component vectors  $V = \sum_i V_i e_i$  are represented in the memory by a set of the oligomers  $E_i$  and  $\underline{E}_i$  that are a subset of all single-stranded oligomers and are in 1:1 correspondence with the basis vectors  $e_i$  for  $i = 1, 2, \dots, m$  in an abstract m-dimensional vector space;

wherein oligomers  $E_i$  and  $\underline{E}_i$  have complementary nucleotide sequences, with  $E_i$  oligomers representing the i-th component of  $V$  for which the amplitude  $V_i$  is positive, and  $\underline{E}_i$  representing the i-th component of  $V$  for which  $V_i$  is negative; and

wherein the concentration of each of oligomers  $E_i$  and  $\underline{E}_i$  is proportional to the absolute value of the amplitude  $V_i$  of the i-th component of  $V$ ;

the method comprising:

(a) preparing a content-addressable memory representing memory matrix  $T_{ij}$  in which are stored data sets corresponding to vectors  $V_i^a$  for  $a = 1$  to  $a = n$ , where  $i = 1, 2, \dots, m$ ,

comprising obtaining for each vector  $V^a$  a set of single-stranded oligomers, each of which comprises a first single-stranded oligomer sequence selected from the group consisting of  $E_i$  or  $\underline{E}_i$  for each i-th component of  $V^a$  for  $i = 1$  to  $i = m$ , and further comprises a second single-

stranded oligomer sequence selected from the group consisting of  $E_j$  or  $\underline{E}_j$  for each  $j$ -th component of  $V^a$  for  $j = 1$  to  $j = m$ , except for  $i = j$ ; and then pooling said sets of dimeric oligomers obtained for each vector  $V^a$  for  $a = 1$  to  $a = n$  thereby forming a set of oligomers representing a content-addressable memory;

(b) combining said pool of dimeric oligomers with a set of oligomers representing partial data set  $U_i^b$  under conditions wherein oligomer sequences  $E_i^b$  and  $\underline{E}_i^b$  of data set  $U_i^b$  hybridize specifically to complementary sequences  $E_j$  and  $\underline{E}_j$  present in said memory pool oligomers; and

obtaining an isolated set of monomeric oligomer strands  $X_i$  comprising the oligomer sequences  $E_i$  and  $\underline{E}_i$  of said memory pool oligomers that hybridized specifically to said  $U_i^b$  oligomers, wherein said  $X_i$  oligomers do not further comprise said  $E_j$  and  $\underline{E}_j$  sequences of said memory pool oligomers that are complementary to said  $U_i^b$  oligomers;

(c) combining said set of  $X_i$  oligomers with a set of single-stranded oligomers comprising a complete, sub-stoichiometric set of  $E_i$  and  $\underline{E}_i$  so that complementary sequences hybridize to each other, denaturing the resulting duplex molecules, and isolating the subset of  $X_i$  oligomer that hybridized specifically to said  $E_i$  and  $\underline{E}_i$  sequences, to obtain a set of saturated  $X_i$  strands,  $S(X_i)$ ;

(d) repeating steps (b) and (c) iteratively, using the set of saturated  $X_i$  strands,  $S(X_i)$  obtained in each previous implementation of step (c) as the set of oligomers representing partial data set  $U_i^b$  employed in the subsequent implementation of step (b), to obtain a set of oligomer strands  $X_i$  produced by step (b) that represents data set  $V_i^b$ .

18. The method of claim 17, wherein the oligomers independently comprise subunits

selected from the group consisting of deoxyribonucleotides, ribonucleotides, and analogs of deoxyribonucleotides or ribonucleotides; and any single oligomer comprises one or a

combination of two or more of said different types of subunits.

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19. The method of claim 17 wherein each of said oligomers forming said content-addressable memory matrix  $T_{ij}$  comprises, in order from the 5' end to the 3' end, (a) an oligomer strand comprising a nucleotide sequence representing an  $i$ -th component of  $V$  selected from the group consisting of  $E_i$  and  $\underline{E}_i$  for  $i = 1$  to  $i = m$ , (b) an oligomer strand comprising a nucleotide sequence representing a  $j$ -th component of  $V$  selected from the group consisting of  $E_j$  and  $\underline{E}_j$  for  $j = 1$  to  $j = m$ , wherein  $j \neq i$ , and (c) a nucleotide sequence  $F$  that is not complementary to any sequence  $E_i$  or  $\underline{E}_i$  for  $i = 1$  to  $i = m$ .

20. The method of claim 19 wherein said oligomers forming said content-addressable memory  $T_{ij}$  are obtained by a method analogous to finding the outer product matrix  $\sum_a V_i^a V_j^a$  comprising, for each  $V^a$ ,

- (a) obtaining a first set of oligomers  $E_i$  and  $\underline{E}_i$  for  $i = 1$  to  $i = m$  representing data elements to be stored in memory;
- (b) obtaining a second set of oligomers  $E_i$  and  $\underline{E}_i$  for  $i = 1$  to  $i = m$  representing data elements to be stored in memory, and comprising at their 3' ends an oligomer sequence  $F$  that is not complementary to any sequence  $E_i$  or  $\underline{E}_i$  for  $i = 1$  to  $i = m$ ;
- (c) combining said first and second sets of oligomers in the presence of ligase so that the 3' ends of said first set of oligomers are ligated to the 5' ends of said second set of oligomers;
- (d) removing from the set of ligated oligomers produced in step (c) those oligomers comprising, in the same oligomer, first and second oligomer sequences that are the same sequence, or that are complementary sequences,



and then pooling said sets of dimeric oligomers obtained for each vector  $V^a$  to yield a set of oligomers representing said content-addressable memory matrix  $T_{ij}$ .

21. The method of claim 17 wherein said set of oligomer strands  $X_i$  is obtained by a method analogous to finding a matrix inner product  $\sum_j T_{ij} U_i^b$  comprising

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(a) obtaining a pool of oligomers forming said content-addressable memory, wherein each oligomer comprises, in order from the 5' end to the 3' end, (i) an oligomer strand comprising a nucleotide sequence representing an  $i$ -th component of  $V$  selected from the group consisting of  $E_i$  and  $\underline{E}_i$  for  $i = 1$  to  $i = m$ , (ii) an oligomer strand comprising a nucleotide sequence representing a  $j$ -th component of  $V$  selected from the group consisting of  $E_j$  and  $\underline{E}_j$  for  $j = 1$  to  $j = m$ , wherein  $j \neq i$ , and (iii) a nucleotide sequence  $F$  that is not complementary to any sequence  $E_i$  or  $\underline{E}_i$  for  $i = 1$  to  $i = m$ ;

(b) obtaining a set of oligomers  $U_i$  representing data vector  $U_i^b$  comprising oligomer sequences of the form  $E_i$  and  $\underline{E}_i$  having concentrations proportional to the corresponding amplitudes in  $U_i^b$ ;

(c) obtaining a set of oligomers  $\underline{U}_i$  comprising oligomer sequences complementary to the oligomer sequences  $E_i$  and  $\underline{E}_i$  present in  $U_i$ , and having concentrations proportional to the corresponding amplitudes of their complements in  $U_i^b$ ;

(d) ligating to the 5' ends of said oligomers  $U_i$  and  $\underline{U}_i$  an oligomer strand  $G$  that comprises at its 3' end an oligomer sequence complementary to said oligomer sequence  $F$ ;

(e) combining said oligomers comprising  $G + \underline{U}_i$  sequences with a sample of said memory pool oligomers so that oligomers comprising  $\underline{U}_i$  sequences complementary to  $E_j$  or  $\underline{E}_j$  sequences in said memory pool oligomers hybridize thereto and form double-stranded

oligomeric structures comprising a restriction enzyme cleavage site between said  $E_j$  or  $\underline{E}_j$  sequence and an oligomer comprising an  $E_i$  or  $\underline{E}_i$  sequence;

cleaving said site with a restriction enzyme; and

isolating the oligomers comprising  $E_i$  and  $\underline{E}_i$  sequences cleaved from said memory pool oligomers hybridizing to said  $G + \underline{U}_i$  oligomers to obtain a set of oligomers  $\{X_i\}$  representing the unchanged sign contribution to the inner product;

(f) combining said oligomers comprising  $G + U_i$  sequences with a sample of said memory pool oligomers having 3' ends that are modified to inhibit polymerase-catalyzed addition of nucleotides at said 3' ends, so that oligomers comprising  $U_i$  sequences complementary to  $E_j$  or  $\underline{E}_j$  sequences in said memory pool oligomers hybridize thereto and form double-stranded oligomeric structures;

performing polymerase-catalyzed extension of the 3' ends of said hybridized  $U_i$  oligomer sequences, using single-stranded  $E_i$  and  $\underline{E}_i$  sequences extending from said double-stranded structures as template strands, thereby generating oligomer sequences complementary to said  $E_i$  and  $\underline{E}_i$  sequences attached to the 3' ends of said  $U_i$  oligomer sequences;

denaturing the resulting double-stranded oligomeric structures;

isolating said  $G + U_i$  oligomers comprising said newly synthesized oligomer sequences complementary to said  $E_i$  and  $\underline{E}_i$  sequences in said memory pool oligomers;

cleaving oligomers comprising said newly synthesized oligomer sequences complementary to  $E_i$  and  $\underline{E}_i$  and said  $G + U_i$  sequences to separate said newly synthesized oligomer sequences complementary to  $E_i$  and  $\underline{E}_i$  from said  $G + U_i$  sequences;

isolating said oligomers comprising sequences complementary to  $E_i$  and  $\underline{E}_i$  in said memory pool oligomers to obtain a set of oligomers  $\{X_i\}$  representing the changed sign

contribution to the inner product; and

- (g) combining in an operation analogous to vector addition an amount of said set of oligomers  $\{X_i\}$  from step (e) above with an equal amount of said set of oligomers  $\{X_i\}$  from step (f) above, to yield a set of oligomer strands  $X_i$  corresponding to the matrix inner product.

22. The method of claim 17 wherein said single-stranded oligomers comprising a complete, sub-stoichiometric set of  $E_i$  and  $\underline{E}_i$  are anchored to a solid support.

23. The method of claim 22 wherein said solid support is contained in a chromatographic column.

24. The method of claim 24 wherein said solid support is, or is attached to, a silicon or  $Al_2O_3$  chip.

25. A content-addressable memory representing a memory matrix  $T_{ij}$  in which are stored data sets corresponding to vectors  $V_i^a$  for  $i = 1$  to  $i = m$ , wherein data elements in the form of m-component vectors  $V = \sum_i V_i e_i$  are each represented in the memory by a set of the oligomers  $E_i$  and  $\underline{E}_i$  that are a subset of all single-stranded oligomers and are each in 1:1 correspondence with the basis vectors  $e_i$  for  $i = 1, 2, \dots, m$  in an abstract m-dimensional vector space;

wherein oligomers  $E_i$  and  $\underline{E}_i$  have complementary nucleotide sequences, with  $E_i$  oligomers representing the i-th component of  $V$  for which the amplitude  $V_i$  is positive, and  $\underline{E}_i$  representing the i-th component of  $V$  for which  $V_i$  is negative; and

wherein the concentration of each of oligomers  $E_i$  and  $\underline{E}_i$  is proportional to the magnitude of the amplitude  $V_i$  of the  $i$ -th component of  $V$ ; comprising:

a content-addressable memory representing memory matrix  $T_{ij}$  in which are stored data sets corresponding to vectors  $V_i^a$  for  $a = 1$  to  $a = n$ , where  $i = 1, 2, \dots, m$ ,

comprising a pool of dimeric, single-stranded oligomers comprising a set of dimeric oligomers for each vector  $V^a$ ,

wherein each oligomer in the set of oligomers for each vector  $V^a$  comprises a first single-stranded oligomer sequence selected from the group consisting of  $E_i$  or  $\underline{E}_i$  for each  $i$ -th component of  $V^a$  for  $i = 1, 2, \dots, m$ , and further comprises a second single-stranded oligomer sequence selected from the group consisting of  $E_j$  or  $\underline{E}_j$  for each  $j$ -th component of  $V^a$  for all  $j = 1$  to  $j = m$ , except for  $i = j$ .

26. The content-addressable memory of claim 25, wherein each of said oligomers forming said content-addressable memory comprises, in order from the 5' end to the 3' end, (a) an oligomer strand comprising a nucleotide sequence representing an  $i$ -th component of  $V$  selected from the group consisting of  $E_i$  and  $\underline{E}_i$  for  $i = 1$  to  $i = m$ , (b) an oligomer strand comprising a nucleotide sequence representing a  $j$ -th component of  $V$  selected from the group consisting of  $E_j$  and  $\underline{E}_j$  for  $j = 1$  to  $j = m$ , wherein  $j \neq i$ , and (c) a nucleotide sequence  $F$  that is not complementary to any sequence  $E_i$  or  $\underline{E}_i$  for  $i = 1$  to  $i = m$ .